Exercise 1.1

1. Write the following quadratic equations in the standard form and point out pure quadratic equations.

i. (x+7) (x-3) = -7

Solution:

$$(x+7)(x-3)=-7$$
$$x(x-3)+7(x-3)=-7$$
$$x^2-3x+7x-21=-7$$
$$x^2+4x-21+7=0$$
$$x^2+4x-14=0$$

The above equation is a quadratic equation.

ii. $\dfrac{x^2+4}{3}-\dfrac{x}{7}=1$

Solution:

$$\frac{x^2+4}{3}-\frac{x}{7}=1$$

Multiply both sides by 21, we get

$$21\times\frac{x^2+4}{3}-21\times\frac{x}{7}=1\times 21$$
$$7(x^2+4)-3x=21$$
$$7x^2+28-3x=21$$
$$7x^2-3x+28-21=0$$
$$7x^2-3x+7=0$$

The above equation is a pure quadratic equation.

iii. $\frac{x}{x+1}+\frac{x}{x+1}=6$

Solution:

$$\frac{x}{x+1}+\frac{x}{x+1}=6$$

$$\frac{x^2+\left(x^2+1\right)^2}{x(x+1)}=6$$

$$x^2+x^2+2x+1=6x(x+1)$$

$$2x^2+2x+1=6x^2+6x$$

$$2x^2-6x^2+2x-6x+1=0$$

$$-4x^2-4x+1=0$$

$$-\left(4x^2+4x-1\right)=0$$

$$\Rightarrow 4x^2+4x-1$$

The above equation is a quadratic equation.

iv. $\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$

Solution:

$$\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$$

$$\frac{x(x+4)-(x-2)^2+4x(x-2)}{x(x-2)}$$

$$\left(x^2+4x\right)-\left(x^2-4x+4\right)+4\left(x^2-8x\right)=0$$

$$x^2+4x-x^2+4x-4+4x^2-8x=0$$

$$x^2-x^2+4x^2+4x+4x-8x-4=0$$

$$4x^2+8x-8x-4=0$$

$$4x^2-4=0$$

$$4\left(x^2-4\right)=0$$

$$x^2-1=0$$

The above equation is a pure quadratic equation.

v. $\frac{x+3}{x+4}-\frac{x-5}{x}=1$

Solution:

$$\frac{x+3}{x+4}-\frac{x-5}{x}=1$$

$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$

$$\left(x^2+3x\right)-x(x-5)-4(x-5)=x(x+4)$$

$$x^2+3x-x^2+5x-4x+20=x^2+4x$$

$$x^2-x^2+3x+5x-4x+20=x^2+4x$$

$$4x+20=x^2+4x$$

$$-x^2+4x-4x+20=0$$

$$-x^2-20=0$$

$$-\left(x^2-20\right)=0$$

$$x^2-20=0$$

The above equation is a pure quadratic equation.

vi. $\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$

Solution:

$$\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$$

$$\frac{(x+1)(x+3)+(x+2)^2}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{x(x+3)+1(x+3)+\left(x^2+4x+4\right)}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{x^2+3x+x+3+x^2+4x+4}{x^2+3x+2x+6}=\frac{25}{12}$$

$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$

$$25\left(x^2+5x+6\right)=12\left(2x^2+8x+7\right)$$

$$25x^2+125x+150=24x^2+96x+84$$

$$x^2+29x+66=0$$

The above equation is a pure quadratic equation.

Q2. Solve by factorization:

i. $x^2-x-20=0$

Solution:

$$x^2-x-20=0$$

$$x^2-5x+4x-20=0$$

$$x(x-5)+4(x-5)=0$$

$$(x+4)(x-5)=0$$

ii. $3y^2=y(y-5)$

Solution:

$$3y^2=y(y-5)$$

$$3y^2=y^2-5y$$

$$3y^2-y^2+5y=0$$

$$2y^2+5y=0$$

$$y(2y+5)=0$$

Either $y=0$ or $2y+5=0$

$$2y=-5$$

$$y=\frac{-5}{2}$$

$$\text{Thus, solution set}=\left\{0,-\frac{5}{2}\right\}$$

iii. $4-32x=17x^2$

Solution:

$$4-32x=17x^2$$

or $17x^2+32x-4=0$

$$17x^2+34x-2x-4=0$$
$$17x(x+2)-2(x+2)=0$$
$$(17x-2)(x+2)=0$$

Either $17x-2=0$ *or* $x=2=0$

$17x=2$ $x=-2$

$x=\dfrac{2}{17}$

Thus, solution set $=\left\{\dfrac{2}{17},-2\right\}$

iv. $x^2-11x=152$

Solution:

$$x^2-11x=152$$
$$x^2-11x-152=0$$
$$x^2-19x+8x-152=0$$
$$x(x-19)+8(x-19)=0$$
$$(x+8)(x-19)=0$$

Either $(x+8)=0$ *or* $(x-19)=0$

$x=-8$ $x=19$

Thus, solution set = {-8,19}

v. $\dfrac{x+1}{x}+\dfrac{x}{x+1}=\dfrac{25}{12}$

Solution:

$$\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$$

$$\frac{(x+1)^2+x^2}{x(x+1)}=\frac{25}{12}$$

$$\frac{x^2+2x+1+x^2}{x^2+x}=\frac{25}{12}$$

$$\frac{2x^2+2x+1}{x^2+x}=\frac{25}{12}$$

$$25(x^2+x)=12(2x^2+2x+1)$$

$$25x^2+25x=24x^2+24x+12$$

$$25x^2-24x^2+25x-24x-12=0$$

$$x^2+x-12=0$$

$$x^2+4x-3x-12=0$$

$$x(x+4)-3(x+4)=0$$

$$(x-3)(x+4)=0$$

Either $x-3=0$ *or* $x+4=0$

$x=3$ $\quad$ $x=-4$

Thus, solution set = $\{3,-4\}$

vi. $$\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$$

Solution:

$$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$$

$$\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$$

$$\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-3)(x-4)}$$

$$\frac{2}{x-9}=\frac{x-4-x+3}{x^2-7x+12}$$

$$2(x^2-7x+12)=-1(x-9)$$

$$2x^2-14x+x+24-9=0$$

$$2x(x-5)-3(x-5)=0$$

$(2x-3)(x-5)=0$

$$\textit{Either } 2x-3=0 \textit{ or } x-5=0$$
$$2x=3 \qquad x=5$$
$$x=\frac{3}{2} \qquad x=5$$

Thus solution set $=\{5,\frac{3}{2}\}$

Q3. Solve the following equation by completing square:

i. $7x^2+2x-1=0$

Solution:

$$7x^2+2x-1=0$$
$$7x^2+2x=1$$
$$\frac{7x^2}{7}+\frac{2x}{7}=\frac{1}{7}$$
$$x^2+\frac{2x}{7}=\frac{1}{7}$$
$$(x^2)+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$$
$$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$$
$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Takin square root on both sides, we get

$$x+\frac{1}{7}=\pm\sqrt{\frac{8}{49}}$$
$$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$$
$$x=\frac{1}{7}\pm\frac{2\sqrt{2}}{7}$$
$$x=\frac{-1\pm2\sqrt{2}}{7}$$

Thus, solution set = $\left\{\frac{-1\pm 2\sqrt{2}}{7}\right\}$

ii. $ax^2+4x-a=0$

Solution:

$ax^2+4x=a$

$\frac{ax^2}{a}+\frac{4x}{a}=\frac{a}{a}$

$x^2+\frac{4x}{a}=1$

$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$

$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$

$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$

Taking square root on both sides, we get

$x+\frac{2}{a}=\pm\sqrt{\frac{a^2+4}{a^2}}$

$x=-\frac{2}{a}\pm\sqrt{\frac{a^2+4}{a^2}}$

$x=\frac{-2\pm\sqrt{a^2+4}}{a}$

Thus, solution set = $\left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

iii. $11x^2-34x+3=0$

Solution:

$11x^2-34x+3=0$

$11x^2-34x=-3$

$$\frac{11x^2}{11}-\frac{34x}{11}=-\frac{3}{11}$$

$$x^2-\frac{34}{11}x=-\frac{3}{11}$$

$$\left(x^2\right)-2(x)\left(\frac{34}{22}\right)+\left(\frac{34}{22}\right)^2=-\frac{3}{11}+\left(\frac{34}{22}\right)^2$$

$$\left(x-\frac{34}{22}\right)^2=-\frac{3}{11}+\frac{1156}{484}$$

$$\left(x-\frac{34}{22}\right)^2=\frac{132+1156}{484}$$

$$\left(x-\frac{34}{22}\right)^2=\frac{1024}{484}$$

Taking square root on both sides we get

$$\left(x-\frac{34}{22}\right)=\pm\sqrt{\frac{1024}{484}}$$

$$x-\frac{34}{22}=\pm\frac{32}{22}$$

$$x=\frac{34}{22}\pm\frac{32}{22}$$

$$x=\frac{34\pm 32}{22}$$

$$x=\frac{34+32}{22},\quad x=\frac{34-32}{22}$$

$$=\frac{66}{22}\qquad =\frac{2}{22}$$

$$=3\qquad =\frac{1}{11}$$

Thus solution set $\left\{3,\frac{1}{11}\right\}$

iv. $lx^2-mx+n=0$

Solution:

$$lx^2-mx+n=0$$

$$lx^2 + mx = -n$$

$$\frac{lx^2}{l} + \frac{mx}{l} = -\frac{n}{l}$$

$$x^2 + \frac{mx}{l} = -\frac{n}{l}$$

$$\left(x^2\right) + 2(x)\left(\frac{m}{2l}\right) + \left(\frac{m}{2l}\right)^2 = -\frac{n}{l} + \left(\frac{m}{2l}\right)^2$$

$$\left(x + \frac{m}{2l}\right)^2 = -\frac{n}{l} + \frac{m^2}{4l^2}$$

$$\left(x + \frac{m}{2l}\right)^2 = \frac{-4\ln + m^2}{4l^2}$$

$$\left(x + \frac{m}{2l}\right)^2 = \frac{m^2 - 4\ln}{4l^2}$$

Taking square root on both sides, we get

$$\sqrt{\left(x + \frac{m}{2l}\right)^2} = \pm\sqrt{\frac{m^2 - 4\ln}{4l^2}}$$

$$x + \frac{m}{2l} = \pm\sqrt{\frac{m^2 - 4\ln}{4l^2}}$$

$$x = -\frac{m}{2l} \pm \sqrt{\frac{m^2 - 4\ln}{4l^2}}$$

$$x = \frac{-m\sqrt{m^2 - 4\ln}}{2l}$$

Thus solution set $= \left\{\frac{-m\sqrt{m^2 - 4\ln}}{2l}\right\}$

v. $3x^2 + 7x = 0$

Solution:

$$3x^2 + 7x = 0$$

$$\frac{3x^2}{3}+\frac{7x}{3}=\frac{0}{3}$$

$$x^2+\frac{7}{3}x=0$$

$$\left(x^2\right)+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=0+\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\pm\sqrt{\left(\frac{7}{6}\right)^2}$$

$$x+\frac{7}{6}=\pm\frac{7}{6}$$

$$x=-\frac{7}{6}\pm\frac{7}{6}$$

$$x=-\frac{7}{6}+\frac{7}{6} \quad or \quad x=\frac{7}{6}+\frac{7}{6}$$

$$x=0 \qquad\qquad x=-\frac{14}{6}$$

$$x=-\frac{7}{3}$$

Thus solution set = $\left\{0,-\frac{7}{3}\right\}$

vi. $x^2-2x-195=0$

Solution:

$$x^2-2x-195=0$$

$$x^2-2x=195$$

$$\left(x^2\right)-2(x)(1)+(1)^2=195+(1)^2$$

$$(x-1)^2=195+1$$

$$(x-1)^2=196$$

Taking square root on both sides

$$\sqrt{(x-1)^2} = \pm\sqrt{196}$$

$$x-1 = \pm 14$$

$$x = 1 \pm 14$$

$$x = 1+14 \qquad x = 1-14$$

$$= 15 \qquad\qquad = -13$$

Thus, solution set = $\{-13, 15\}$

vii. $-x^2 + \dfrac{15}{2} = \dfrac{7}{2}x$

Solution:

$$-x^2 - \frac{7}{2}x = \frac{15}{2}$$

$$-\left(x^2 + \frac{7}{2}x\right) = -\frac{15}{2}$$

$$x^2 + \frac{7}{2}x = \frac{15}{2}$$

$$(x)^2 + 2(x)\left(\frac{7}{4}\right) + \left(\frac{7}{4}\right)^2 = \frac{15}{2} + \left(\frac{7}{4}\right)^2$$

$$\left(x+\frac{7}{4}\right)^2 = \frac{15}{2} + \frac{49}{16}$$

$$\left(x+\frac{7}{4}\right)^2 = \frac{120+49}{16}$$

$$\left(x+\frac{7}{4}\right)^2 = \frac{169}{16}$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{7}{4}\right)^2} = \pm\sqrt{\frac{169}{16}}$$

$$x + \frac{7}{4} = \pm\frac{13}{4}$$

$$x = -\frac{7}{4} + \frac{13}{4} \qquad x = -\frac{7}{4} - \frac{13}{4}$$

$x=\frac{3}{2} \qquad x=-5$

Thus, solution set = $\left\{\frac{3}{2},-5\right\}$

viii. $x^2+17x+\frac{33}{4}=0$

Solution:

$x^2+17x=-\frac{33}{4}$

$(x)^2=2(x)\left(\frac{17}{2}\right)+\left(\frac{17}{2}\right)^2=-\frac{33}{4}+\left(\frac{17}{2}\right)^2$

$\left(x+\frac{17}{2}\right)^2=-\frac{33}{4}+\frac{289}{4}$

$\left(x+\frac{17}{2}\right)^2=\frac{256}{4}$

Taking square root on both sides,

$\sqrt{\left(x+\frac{17}{2}\right)^2}=\pm\sqrt{\frac{256}{4}}$

$x+\frac{17}{2}=\pm\frac{16}{2}$

$x=-\frac{17}{2}\pm\frac{16}{2}$

$x=-\frac{17}{2}+\frac{16}{2} \qquad x=-\frac{17}{2}-\frac{16}{2}$

Thus solution set = $\left\{-\frac{1}{2},-\frac{33}{2}\right\}$

ix. $4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$

Solution:

$$4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$$
$$\frac{4(3x+1)-8}{3x+1}=\frac{3x^2+5}{3x+1}$$
$$\frac{12x+4-8}{3x+1}=\frac{3x^2+5}{3x+1}$$
$$\frac{12x-4}{3x+1}=\frac{3x^2+5}{3x+1}$$

Multiplying both sides by $(3x+1)$, we get

$$12x-4=3x^2+5$$
$$or\ 3x^2+5-12x+4=0$$
$$3x^2-12x+9=0$$
$$3(x^2-4x+3)=0$$
$$\Rightarrow x^2-4x+3=0$$
$$x^2-3x-x+3=0$$
$$x(x-3)-1(x-3)=0$$
$$(x-1)(x-3)=0$$

Either $x-1=0$ *or* $x-3=0$

$x=1$ $\quad$ $x=3$

Thus solution set $=\{1,3\}$

x. $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution:

$$7(x+2a)^2+3a^2=5a(7x+23a)$$
$$7(x^2+4ax+4a^2)+3a^2=35ax+115a^2$$
$$7x^2+28ax+28a^2+3a^2=35ax+115a^2$$
$$7x^2-7ax-84a^2=0$$
$$7(x^2-ax-12a^2)=0$$
$$x^2-ax-12a^2=0$$

$$x^2 - ax = 12a^2$$

$$(x)^2 = 2(x)\left(\frac{a}{2}\right) + \left(\frac{a}{2}\right)^2 = 12a^2 + \left(\frac{a}{2}\right)^2$$

$$\left(x - \frac{a}{2}\right)^2 = 12a^2 + \frac{a^2}{4}$$

$$\left(x - \frac{a}{2}\right)^2 = \frac{49a^2}{4}$$

Taking square root on both sides, we get

$$\sqrt{\left(x - \frac{a}{2}\right)^2} = \pm\sqrt{\frac{49a^2}{4}}$$

$$x - \frac{a}{2} = \pm\frac{7a}{2}$$

$$x = \frac{a}{2} \pm \frac{7a}{2}$$

$$x = \frac{a}{2} + \frac{7a}{2} \qquad x = \frac{a}{2} - \frac{7a}{2}$$

$$= \frac{8a}{2} \qquad = -\frac{6a}{2}$$

$$= 4a \qquad = -3a$$

Thus solution set = $\{-3a, 4a\}$

Quadratic Formula:

Derivation of quadratic formula by using completing square method.

The quadratic equation in standard form is

$$ax^2 + bx + c = 0,\ a \neq 0$$

Dividing each term o the equation by a, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Shifting constant term $\frac{c}{a}$ to the right, we have

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides, we obtain

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

or

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$or\ x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}\Rightarrow x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Thus, x= $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ is known as "quadratic formula".

Exercise 1.2

Q1. Solve the following equations using quadratic formula:

(i) $2-x^2=7x$

Solution:

$$2-x^2=7x$$
$$-x^2-7x+2=0$$
$$-\left(x^2+7x-2\right)=0$$
$$\Rightarrow x^2+7x-2=0$$

Compare it with quadratic equations, we have

$ax^2+bx+c=0$

Here a=1, b=7, c=-2

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-7\pm\sqrt{(7)^2-4(1)(-2)}}{2(1)}$$
$$x=\frac{-7\pm\sqrt{49+8}}{2}$$
$$x=\frac{-7\pm\sqrt{57}}{2}$$

$\text{Thus, solution set}=\left\{\dfrac{-7\pm\sqrt{57}}{2}\right\}$

(ii) $5x^2+8x+1=0$

Solution:

$$5x^2+8x+1=0$$

Compare it with quadratic equation, we have

$ax^2 + bx + c = 0$

Here $a = 5, b = 8, c = 1$

Now $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{2\left(-4 \pm \sqrt{11}\right)}{10}$$

$$x = \frac{-4 \pm \sqrt{11}}{5}$$

Thus solution set $= \left\{\dfrac{-4 \pm \sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$

Solution:

$\sqrt{3}x^2 + x = 4\sqrt{3}$

$\sqrt{3}x^2 + x - 4\sqrt{3} = 0$

Compare it with quadratic equation, we have

$ax^2 + bx + c = 0$

Here $a = \sqrt{3}, b = 1, c = -4\sqrt{3}$

Now $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4\left(\sqrt{3}\right)\left(-4\sqrt{3}\right)}}{2\left(\sqrt{3}\right)}$$

$$x=\frac{-1\pm\sqrt{1-48}}{2\left(\sqrt{3}\right)}$$

$$x=\frac{-1\pm\sqrt{49}}{2\sqrt{3}}$$

$$x=\frac{-1\pm 7}{2\sqrt{3}}$$

$$x=\frac{-1+7}{2\sqrt{3}} \qquad x=\frac{-1-7}{2\sqrt{3}}$$

$$x=\frac{6}{2\sqrt{3}} \qquad x=\frac{-8}{2\sqrt{3}}$$

$$x=\frac{3}{\sqrt{3}} \qquad x=-\frac{4}{\sqrt{3}}$$

$$x=\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$x=\frac{3\sqrt{3}}{\left(\sqrt{3}\right)^2}$$

$$x=\frac{3\sqrt{3}}{3}$$

$$x=\sqrt{3}$$

Thus solution set = $\left\{\sqrt{3},-\frac{4}{\sqrt{3}}\right\}$

(iv) $4x^2-14=3x$

Solution:

$4x^2-14=3x$

$4x^2-3x-14=0$

Compare it with, we have

$ax^2+bx+c=0$

Here $a=4,b=-3,c=-14$

Now $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-3) \pm \sqrt{(3)^2 - 4(4)(-14)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{9 + 224}}{8}$$

$$x = \frac{3 \pm \sqrt{233}}{8}$$

$$x = \frac{3 \pm \sqrt{233}}{8}$$

Thus solution set = $\left\{ \frac{3 \pm \sqrt{233}}{8} \right\}$

(v) $6x^2 - 3 - 7x = 0$

Solution:

Comparing the given equation, we have

$ax^2 + bx + c = 0$

Here $a = 6, b = -7, c = -3$

Now $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$x = \frac{7 + 11}{12}$ $\quad$ $x = \frac{7 - 11}{12}$

$x = \frac{18}{12}$ $\quad$ $x = -\frac{1}{3}$

$x = \frac{3}{2}$ $\quad$ $x = -\frac{1}{3}$

Thus solution set = $\left\{-\frac{1}{3}, \frac{3}{2}\right\}$

(vi) $3x^2 + 8x + 2 = 0$

Solution:

$3x^2 + 8x + 2 = 0$

Compare it with, we have

$ax^2 + bx + c = 0$

Here $a = 3, b = 8, c = 2$

Now $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$

$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$

$x = \frac{-8 \pm \sqrt{40}}{6}$

$x = \frac{-8 \pm \sqrt{40}}{6}$

$x = \frac{-8 \pm \sqrt{10}}{6}$

$x = \frac{2\left(-4 \pm \sqrt{10}\right)}{6}$

$x = \frac{\left(-4 \pm \sqrt{10}\right)}{3}$

Thus solution set = $\left\{\frac{\left(-4 \pm \sqrt{10}\right)}{3}\right\}$

(vii) $\frac{3}{x-6}-\frac{4}{x-5}=1$

Solution:

$$\frac{3}{x-6}-\frac{4}{x-5}=1$$
$$\frac{3(x-5)-4(x-6)}{(x-6)(x-5)}=1$$
$$3x-15-4x+24=(x-6)(x-5)$$
$$-x+9=x^2-11x+30$$
$$x^2-11x+x+30-9=0$$
$$x^2-10x+21=0$$

Compare it with, we have

$ax^2+bx+c=0$

Here $a=1, b=-10, c=21$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(3)}$$
$$x=\frac{10\pm\sqrt{100-84}}{2}$$
$$x=\frac{10\pm\sqrt{16}}{2}$$
$$x=\frac{-8\pm 4}{2}$$

$x=\frac{10+4}{2}$ $\quad$ $x=\frac{10-4}{2}$

$x=\frac{14}{2}$ $\quad$ $x=\frac{6}{2}$

$x=7$ $\quad$ $x=3$

Thus, solution set = $\{3,7\}$

(viii) $\frac{x+2}{x+1}-\frac{4-x}{2x}=2\frac{1}{3}$

Solution:

$$\frac{x+2}{x+1}-\frac{4-x}{2x}=2\frac{1}{3}$$

$$\frac{2x(x+2)-(x-1)(4-x)}{2x(x-1)}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(4x-x^2-4+x)}{2x^2-2x}=\frac{7}{3}$$

$$\frac{2x^2+4x+x^2-5x+4}{2x^2-2x}=\frac{7}{3}$$

$$7(2x^2-2x)=3(3x^2-x+4)$$

$$14x^2-14x=9x^2-3x+12$$

$$14x^2-9x^2-14x+3x-12=0$$

$$5x^2-11x-12=0$$

Compare it with, we have

$$ax^2+bx+c=0$$

Here $a=5, b=-11, c=-12$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$$

$$x=\frac{11\pm\sqrt{121-240}}{10}$$

$$x=\frac{11\pm\sqrt{361}}{10}$$

$$x=\frac{11\pm19}{10}$$

$$x=\frac{11+19}{10} \qquad x=\frac{11-10}{10}$$

$$x=\frac{30}{10} \qquad x=\frac{-8}{10}$$

$$x=3 \qquad x=-\frac{4}{5}$$

Thus, solution set = $\left\{3, -\frac{4}{5}\right\}$

(ix) $\frac{a}{x-b}+\frac{b}{x-a}=2$

Solution:

$$\frac{a}{x-b}+\frac{b}{x-a}=2$$
$$\frac{a(x-a)+b(x-b)}{(x-a)(x-b)}=2$$
$$ax-a^2+bx-b^2=2(x-a)(x-b)$$
$$ax+bx-a^2-b^2=2(x^2-ax-bx+ab)$$
$$ax+bx-a^2-b^2=2x^2-2ax-2bx+2ab$$
$$2x^2-3ax-3bx+2ab+a^2+b^2=0$$
$$2x^2-3(a+b)x+(2ab+a^2+b^2)=0$$
$$2x^2-3(a+b)x+(a+b)^2=0$$

Compare it with, we have

$ax^2+bx+c=0$

Here $a=2, b=-3(a+b), c=(a+b)^2$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-[-3(a+b)]\pm\sqrt{[-3(a+b)]^2-4(2)(a+b)^2}}{2(2)}$$
$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$
$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$
$$x=\frac{3(a+b)\pm(a+b)}{4}$$

$$x=\frac{3(a+b)+(a+b)}{4} \qquad x=\frac{3(a+b)-(a+b)}{4}$$

$$x=\frac{3a+3b+a+b}{4} \qquad x=\frac{3a+3b-a-b}{4}$$

$$x=\frac{4a+4b}{4} \qquad x=\frac{2a+2b}{4}$$

$$x=\frac{4(a+b)}{4} \qquad x=\frac{2(a+b)}{4}$$

$$x=a+b, \qquad x=\frac{1}{2}(a+b)$$

Thus, solution set = $\left\{(a+b),\frac{1}{2}(a+b)\right\}$

(x) $-(l+m)-lx^2+(2l+m)x=0$

Solution:

$$-(l+m)-lx^2+(2l+m)x=0$$
$$-lx^2+(2l+m)x-(l+m)=0$$
$$-\left[lx^2-(2l+m)x+(l+m)\right]=0$$
$$lx^2-(2l+m)x+(l+m)=0$$

Compare it with, we have

$ax^2+bx+c=0$

Here $a=l, b=-(2l+m), c=(l+m)$

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-\left[-(2l+m)\pm\sqrt{\left[-(2l+m)\right]^2-4(l)(l+m)^2}\right]}{2l}$$

$$x=\frac{(2l+m)\pm\sqrt{(2l+m)^2-4l(l+m)}}{2l}$$

$$x=\frac{(2l+m)\pm\sqrt{4l^2+4lm+m^2-4l^2-4lm}}{2l}$$

$$x = \frac{(2l+m) \pm \sqrt{m^2}}{2l}$$

$$x = \frac{(2l+m) \pm m}{2l}$$

$$x = \frac{(2l+m)+m}{2l}, \qquad x = \frac{(2l+m)-m}{2l}$$

$$x = \frac{2l+2m}{2l}, \qquad x = \frac{2l+2m-m}{2l}$$

$$x = \frac{2(l+m)}{2l}, \qquad x = \frac{2l}{2l}$$

$$x = \frac{(l+m)}{l}, \qquad x = l$$

Thus, solution set = $\left\{ \frac{(l+m)}{l}, l \right\}$

Exercise 1.3

Q1. Solve the following equations.

(1) $2x^4 - 11x - 5 = 0$

Solution:

$2x^4 - 11x - 5 = 0 \qquad ......(i)$

$Let\, x^2 = y \;\; then\;\; x^4 = y^2$

So eq.(i) becomes

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(2y-1)(y-5) = 0$

$Either\; (2y-1) = 0 \;\; or \;\; (y-5) = 0$

$2y = 1 \qquad\qquad y = 5$

$Put \quad y = \frac{1}{2}$ in $x^2 = y,$ we get $\qquad Put\;\; y = 5 \;\; in \;\; x^2 = y, we\,get$

$x^2 = \frac{1}{2} \qquad\qquad x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}} \qquad\qquad \sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\frac{1}{\sqrt{2}} \qquad\qquad x = \pm\sqrt{5}$

Thus, solution set = $\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

(2) $2x^4 = 9x^2 - 4$

Solution:

$$2x^4 = 9x^2 - 4$$
$$2x^4 - 9x^2 + 4 = 0 \qquad ......(i)$$

$Let\, x^2 = y \;\; then\;\; x^4 = y^2$

So eq.(i) becomes

$$2y^2 - 9y + 4 = 0$$
$$2y^2 - 8y - y + 4 = 0$$
$$2y(y-4) - 1(y-4) = 0$$
$$(2y-1)(y-4) = 0$$

$Either\; (2y-1) = 0 \;\; or \;\; (y-4) = 0$

$$2y = 1 \qquad\qquad y = 4$$

$Put \quad y = \frac{1}{2}$ in $x^2 = y$, we get $\qquad Put\;\; y = 4 \;\; in\;\; x^2 = y, we\, get$

$$x^2 = \frac{1}{2} \qquad\qquad x^2 = 4$$
$$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}} \qquad\qquad \sqrt{x^2} = \pm\sqrt{4}$$
$$x = \pm\frac{1}{\sqrt{2}} \qquad\qquad x = \pm 2$$

Thus, solution set = $\left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

(3) $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution:

$$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$$
$$5x^{\frac{1}{2}} - 7x^{\frac{1}{4}} + 2 = 0 \qquad ......(i)$$

$Let\ x^{\frac{1}{4}} = y\ \ then\ x^{\frac{1}{2}} = y^2$

So eq.(i) becomes

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(5y-2)(y-1) = 0$

$Either\ (5y-2) = 0\ \ or\ \ (y-1) = 0$

$5y = 2 \qquad y = 1$

$y = \frac{2}{5} \qquad y = 1$

Put $y = \frac{2}{5}$ in $x^{\frac{1}{4}} = y$, we get

$x^{\frac{1}{4}} = y$

$x^{\frac{1}{4}} = \frac{2}{5}$

Taking power '4' on both siedes we get

$\left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{5}\right)^4$

$x = \frac{2^4}{5^4}$

$x = \frac{16}{625}$

$Put\ \ y = 1\ \ in\ \ x^{\frac{1}{2}} = y, we\, get$

$x^{\frac{1}{4}} = y$

$x^{\frac{1}{4}} = 1$

Taking power '4' on both siedes we get

$\left(x^{\frac{1}{4}}\right)^4 = (1)^4$

$\left(x^{\frac{1}{4}}\right)^4 = 1$

$x = 1$

Thus, solution set = $\left\{\frac{16}{625}, 1\right\}$

(4) $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

Solution:

$x^{\frac{2}{3}}+54=15x^{\frac{1}{3}}$

$x^{\frac{2}{3}}-15x^{\frac{1}{3}}+54=0 \quad ......(i)$

$Let\, x^{\frac{1}{2}}=y,\ then\ x^{\frac{2}{3}}=y^2$

So eq.(i) becomes

$y^2-15y+54=0$

$y^2-9y-6y+54=0$

$y(y-9)-6(y-9)=0$

$(y-6)(y-9)=0$

$Either\ (y-9)=0\ or\ (y-6)=0$

$y=9 \qquad y=6$

$Put \quad y=9$ in $x^{\frac{1}{3}}=y,$ we get

$x^{\frac{1}{3}}=y$

$x^{\frac{1}{3}}=9$

Taking cube on both siedes we get

$\left(x^{\frac{1}{3}}\right)^3=(9)^3$

$x=729$

$Put\ y=6\ in\ x^{\frac{1}{3}}=y, we\ get$

$x^{\frac{1}{3}}=y$

$x^{\frac{1}{3}}=6$

Taking cube on both siedes we get

$\left(x^{\frac{1}{3}}\right)^3=(6)^3$

$x=216$

Thus, solution set = $\{729,216\}$

(5) $3x^{-2}+5=8x^{-1}$

Solution:

$3x^{-2}+5=8x^{-1}$

$3x^{-2}-8x^{-1}+5=0 \quad ......(i)$

$Let\ x^{-1} = y,\ then\ x^{-2} = y^2$

So eq.(i) becomes

$3y^2 - 8y + 5 = 0$

$3y^2 - 5y - 3y + 5 = 0$

$y(3y-5) - 1(3y-5) = 0$

$(y-1)(3y-5) = 0$

$Either\ y - 1 = 0\ or\ 3y - 5 = 0$

$y = 1 \qquad 3y = 5$

$y = 1 \qquad y = \frac{5}{3}$

$Put\ y = 1$ in $x^{-1} = y$, we get

$x^{-1} = y$

$x^{-1} = 1$

$\frac{1}{x} = 1$

$x = 1$

$Put\ y = \frac{5}{3}\ in\ x^{-1} = y, we\ get$

$x^{-1} = y$

$x^{-1} = \frac{5}{3}$

$\frac{1}{x} = \frac{5}{3}$

$x = \frac{5}{3}$

Thus, solution set = $\left\{1, \frac{5}{3}\right\}$

(6) $\left(2x^2+1\right) + \frac{3}{2x^2+1} = 4$

Solution:

$\left(2x^2+1\right) + \frac{3}{2x^2+1} = 4$

$\left(2x^2+1\right) + \frac{3}{2x^2+1} = 4$ ......(i)

$Let\ 2x^2 + 1 = y,$

So eq.(i) becomes

$$y+\frac{3}{y}=4$$

Multiplying both sides by 'y', we get

$$y^2+3=4y$$
$$y^2-4y+3=0$$
$$y^2-3y-y+3=0$$
$$y(y-3)-1(y-3)=0$$
$$(y-1)(y-3)=0$$

Either $y-1=0$ *or* $y-3=0$

$y=1$ $\qquad$ $y=3$

Put $y=1$ in $2x^2+1=y$, we get

$$2x^2+1=1$$
$$2x^2=1-1$$
$$2x^2=0$$
$$x^2=0$$
$$x=0$$

Put $y=3$ *in* $2x^2+1=y$, *we get*

$$2x^2+1=3$$
$$2x^2=3-1$$
$$2x^2=2$$
$$x^2=1$$
$$x=\pm 1$$

Thus, solution set = $\{-1,0,1\}$

(7) $\dfrac{x}{x-3}+4\left(\dfrac{x-3}{x}\right)=4$

Solution:

$$\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4 \quad ......(i)$$

Let $\dfrac{x}{x-3}=y,$

So eq.(i) becomes

$$y+4\left(\frac{1}{y}\right)=4$$

Multiplying both sides by 'y', we get

$$y^2+4=4y$$
$$y^2-4y+4=0$$
$$(y)^2-2(y)(2)+(2)^2=0$$
$$(y-2)^2=0$$
$$(y-2)=0$$

Put $y=2$ in $\frac{x}{x-3}=y$, we get

$$\frac{x}{x-3}=y$$
$$\frac{x}{x-3}=2$$
$$2(x-3)=x$$
$$2x-6=x$$
$$2x-x=6$$
$$x=6$$

Thus, solution set = $\{6\}$

(8) $\frac{4x+1}{4x-1}+\frac{4x-1}{4x-1}=2\frac{1}{6}$

Solution:

$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x-1}=2\frac{1}{6}$$
$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x-1}=\frac{13}{6} \quad ......(i)$$

Let $\frac{4x+1}{4x-1}=y,$

So eq.(i) becomes

$$y+\frac{1}{y}=\frac{13}{6}$$

Multiplying both sides by '6y', we get

$$6y^2+6=13y$$
$$6y^2-13y+6=0$$
$$6y^2-9y-4y+6=0$$
$$3y(2y-3)-2(2y-3)=0$$

$$(3y-2)(2y-3)=0$$

Either $3y-2=0$ *or* $2y-3=0$

$3y=2 \qquad 2y=3$

$y=\frac{2}{3} \qquad y=\frac{3}{2}$

Put $y=\frac{2}{3}$ in $\frac{4x+1}{4x-1}=y$, we get

$$\frac{4x+1}{4x-1}=y$$
$$\frac{4x+1}{4x-1}=\frac{2}{3}$$
$$3(4x+1)=2(4x-1)$$
$$12x+3=8x-2$$
$$12x-8x=-2-3$$
$$4x=-5$$
$$x=-\frac{5}{4}$$

Put $y=\frac{3}{2}$ in $\frac{4x+1}{4x-1}=y$, we get

$$\frac{4x+1}{4x-1}=y$$
$$\frac{4x+1}{4x-1}=\frac{3}{2}$$
$$3(4x-1)=2(4x+1)$$
$$12x-3=8x+2$$
$$12x-8x=2+3$$
$$4x=5$$
$$x=\frac{5}{4}$$

Thus, solution set = $\left\{\pm\frac{5}{4}\right\}$

(9) $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$

Solution:

$$\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12} \qquad ......(i)$$

$Let\ \frac{x-a}{x+a}=y,$

So eq.(i) becomes

$$y-\frac{1}{y}=\frac{7}{12}$$

Multiplying both sides by '12y', we get

$$12y^2-12=7y$$
$$12y^2-7y-12=0$$
$$12y^2-16y+9y-12=0$$
$$4y(3y-4)+3(3y-4)=0$$
$$(4y+3)(3y-4)=0$$

Either $4y+3=0$ *or* $3y-4=0$

$4y=-3 \qquad 3y=4$

$y=-\frac{3}{4} \qquad y=\frac{4}{3}$

Put $y=-\frac{3}{4}$ in $\frac{x-a}{x+a}=y,$ we get

$$\frac{x-a}{x+a}=y$$
$$\frac{x-a}{x+a}=-\frac{3}{4}$$
$$4(x-a)=-3(x+a)$$
$$4x-4a=-3x-3a$$
$$4x+3x=4a-3a$$
$$7x=a$$
$$x=\frac{a}{7}$$

Put $y=\frac{4}{3}$ in $\frac{x-a}{x+a}=y,$ we get

$$\frac{x-a}{x+a}=y$$
$$\frac{x-a}{x+a}=\frac{4}{3}$$
$$4(x+a)=3(x-a)$$
$$4x+4a=3x-3a$$
$$4x-3x=-4a-3a$$
$$x=-7a$$
$$x=\frac{5}{4}$$

Thus, solution set = $\left\{-7a,\frac{a}{7}\right\}$

(10) $x^4-2x^3-2x^2+2x+1=0$

Solution:

$x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

Dividing each term by x^2, *we get*

$$\frac{x^4}{x^2} - 2\frac{x^3}{x^2} - 2\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2} = 0$$

$$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2x + \frac{2}{x} - 2 = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 2\left(x - \frac{2}{x}\right) - 2 = 0 \quad .....(i)$$

Let $\quad x - \frac{1}{x} = y$

$$\left(x - \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} - 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 + 2$$

So eq. (i) becomes

$$y^2 + 2 - 2y - 2 = 0$$

$$y^2 - 2y = 0$$

$$y(y-2) = 0$$

Either $y = 0$ *or* $y - 2 = 0, \Rightarrow y = 2$

Put $y = 0$ in $x - \frac{1}{x} = y$, we get

$$x - \frac{1}{x} = y$$

$$x - \frac{1}{x} = 0$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x^2 = \pm 1$$

Put $y = 2$ in $x - \frac{1}{x} = y$, we get

$$x - \frac{1}{x} = y$$

$$x - \frac{1}{x} = 2$$

$$x^2 - 1 = 2x$$

$$x^2 - 2x - 1 = 0$$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{4+4}}{2}$$

$$x=\frac{2\pm\sqrt{8}}{2}$$

$$x=\frac{2\pm2\sqrt{2}}{2}$$

$$x=1\pm\sqrt{2}$$

Thus, solution set = $\left\{\pm1, 1\pm\sqrt{2}\right\}$

(11) $2x^4+x^3-6x^3+x+2=0$

Solution:

$$2x^4+x^3-6x^3+x+2=0$$

Dividing each term by x^2, *we get*

$$\frac{2x^4}{x^2}+\frac{x^3}{x^2}-\frac{6x^2}{x^2}+\frac{x}{x^2}+\frac{2}{x^2}=0$$

$$2x^2+x-6+\frac{1}{x}+\frac{2}{x^2}=0$$

$$2x^2+\frac{2}{x^2}+x+\frac{1}{x}-6=0$$

$$2\left(x^2+\frac{2}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0 \quad .....(i)$$

Let $\quad x+\frac{1}{x}=y$

$$\left(x+\frac{1}{x}\right)^2=y^2$$

$$x^2+\frac{1}{x^2}+2=y^2$$

$$x^2+\frac{1}{x^2}=y^2-2$$

So eq. (i) becomes

$$2\left(y^2-2\right)+y-6=0$$

$$2y^2-4+y-6=0$$
$$2y^2+y-10=0$$
$$2y^2+5y-4y-10=0$$
$$y(2y+5)-2(2y+5)=0$$
$$(y-2)(2y+5)=0$$

Either $y-2=0$ *or* $2y+5=0$

$y=2$ $\quad 2y=-5$

$y=2$ $\quad y=\frac{-5}{2}$

Put $y=2$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$
$$x+\frac{1}{x}=2$$
$$x^2+1=2x$$
$$x^2-2x+1=0$$

$$(x-2)^2=0$$
$$\Rightarrow \quad x-1=0$$
$$\Rightarrow \quad x=1$$

Put $y=-\frac{5}{2}$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$
$$x+\frac{1}{x}=-\frac{5}{2}$$
$$x^2+1=2=-5x$$
$$2x^2+5x+2=0$$

$$2x^2+4x+x+2=0$$
$$2x(x+2)+1(x+2)=0$$

Either $2x+1=0$ or $x+2=0$

$2x=-1$ $\quad x=-2$

$x=-\frac{1}{2}$

Thus, solution set = $\left\{1,-2,-\frac{1}{2}\right\}$

(12) $4.2^{2x+1}-9.2^{x}+1=0$

Solution:

$$4.2^{2x+1}-9.2^{x}+1=0$$
$$4.2^{2x+1}.2^{1}-9.2^{x}+1=0 \qquad ....(i)$$

Let $2^x = y$ Then $2^{2x} = y^2$

So eq. (i) becomes

$4.y^2.2^1 - 9.y + 1 = 0$

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$(8y-1)(y-1) = 0$

Either $8y - 1 = 0$ *or* $y - 1 = 0$

$8y = 1 \qquad y = 1$

$y = \frac{1}{8} \qquad y = 1$

Put $y = \frac{1}{8}$ in $2^x = y$, we get

$2^x = y$

$2^x = \frac{1}{8}$

$2^x = \frac{1}{2^3}$

$2^x = 2^3$

Put $y = 1$ in $2^x = y$, we get

$2^x = y$

$2^x = 1$

$2^x = 2 = 0$

$x = 0$

Thus, solution set = $\{-3, 0\}$

(13) $3^{2x+2} = 12.3^x - 3$

Solution:

$3^{2x+2} = 12.3^x - 3$

$3^{2x}.3^2 - 12.3^x + 3 = 0 \qquad \text{.........}(i)$

Let $3^x = y$ Then $3^{2x} = y^2$

So eq. (i) becomes

$y^2.9 - 12y + 3 = 0$

$9y^2 - 12y + 3 = 0$

$9y^2 - 9y - 3y + 3 = 0$

$9y(y-1) - 3(y-1) = 0$

$(9y-3)(y-1) = 0$

Either $9y - 3 = 0$ *or* $y - 1 = 0$

$9y = 3 \qquad y = 1$

$y = \frac{3}{9} \qquad y = 1$

$y = \frac{1}{9} \qquad y = 1$

Put $y = \frac{1}{3}$ in $3^x = y$, we get

$3^x = y$

$3^x = \frac{1}{3}$

$3^x = 3^{-1}$

$x = -1$

Put $y = 1$ in $3^x = y$, we get

$3^x = y$

$3^x = 1$

$3^x = 3^0$

$x = 0$

Thus, solution set = $\{-1, 0\}$

(14) $2^x + 64.2^{-x} - 20 = 0$

Solution:

$2^x + 64.2^{-x} - 20 = 0 \qquad ....(i)$

Let $2^x = y$ Then $2^{-x} = \frac{1}{y}$

So eq. (i) becomes

$y - 64.\frac{1}{y} = 20 = 0$

$y^2 - 64 - 20y = 0$
$y(y-16) - 4(y-16) = 0$
$(y-4)(y-16) = 0$

$y^2 - 20y - 64 = 0$
$y^2 - 16y - 4y - 64 = 0$

Either $y - 4 = 0$ *or* $y - 16 = 0$
$y = 4 \qquad y = 16$

Put $y = 4$ in $2^x = y$, we get
$2^x = y$
$2^x = 4$
$2^x = 2^2$
$x = 2$

Put $y = 16$ in $2^x = y$, we get
$2^x = y$
$2^x = 16$
$2^x = 2^4$
$x = 4$

Thus, solution set = $\{2, 4\}$

(15) $(x+1)(x+3)(x-5)(x-7) = 192$

Solution:

$(x+1)(x+3)(x-5)(x-7) = 192$

As $1 - 5 = 3 - 7$

So $[(x+1)(x-5)][(x+3)(x-7)] = 192$
$[x^2 - 5x + x - 5][x^2 - 7x + 3x - 21] = 192$
$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192 \qquad ......(i)$
$(y-5)(y-21) = 192$
$y^2 - 21y - 5y + 105 = 192$
$y^2 - 26y + 105 - 192 = 0$
$y^2 - 26y - 87 = 0$
$y^2 - 29y + 3y - 87 = 0$

$$(y+3)(y-29)=0$$

Either $y+3=0$ *or* $y-29=0$

$y=-3$ $\qquad$ $y=29$

Put $y=-3$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=-3$$
$$x^2-4x+3=0$$
$$x^2-3x-x+3=0$$
$$x(x-3)-1(x-3)=0$$
$$(x-1)(x-3)=0$$

Either $x-1=0$ *or* $x-3=0$

$x=1$ $\qquad$ $x=3$

Put $y=29$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=29$$
$$x^2-4x-29=0$$

Here $a=1, b=-4, c=-29$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{16+116}}{2}$$
$$x=\frac{4\pm\sqrt{132}}{2}$$
$$x=\frac{4\pm2\sqrt{33}}{2}$$
$$x=\frac{2\left(2\pm\sqrt{33}\right)}{2}$$
$$x=2\pm\sqrt{33}$$

Thus, solution set = $\left\{1,3,2\pm\sqrt{33}\right\}$

(16) $(x-1)(x-2)(x-8)(x+5)360=0$

Solution:

$(x-1)(x-2)(x-8)(x+5)360=0$

As $\quad -1-2=-8+5$

$\qquad -3=-3$

So $\quad [(x-1)(x-2)][(x-8)(x+5)]+360=0$

$[x^2-2x-x+2][x^2+5x-8x-40]+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0 \quad ......(i)$

Let $\quad x^2-3y=y$

So eq.(i) *becomes*

$$(y+2)(y-40)+360=0$$
$$y^2-40y+2y-80+360=0$$
$$y^2-38y+280=0$$
$$y^2-28y-10y+280=0$$
$$y(y-28)-10(y-28)=0$$
$$(y-10)(y-28)=0$$

Either $y-10=0$ *or* $y-28=0$

$\qquad y=10 \qquad y=28$

Put $\quad y=10$ in $x^2-3x=y$, we get

$$x^2-3x=y$$
$$x^2-3x=10$$
$$x^2-3x-10=0$$
$$x^2-5x+2x-10=0$$
$$x(x-5)+2(x-5)=0$$
$$(x+2)(x-5)=0$$

Either $x+2=0$ *or* $x-5=0$

$\qquad x=-2 \qquad x=5$

Put $y=28$ in $x^2-3x=y$, we get

$$x^2-3x=y$$
$$x^2-3x=28$$
$$x^2-3x-28=0$$
$$x^2-7x+4x-28=0$$
$$x(x-7)+4(x-7)=0$$
$$(x+4)(x-7)=0$$

Either $x+4=0$ *or* $x-7=0$

$\qquad x=-4 \qquad x=7$

Thus, solution set = $\{-4,-2,5,7\}$

Radical Equations:

An equation involving expression under the radical sign is called a radical equation

e.g., $\sqrt{x+3} = x+1$ *and* $\sqrt{x-1} = \sqrt{x-2}+1$

Exercise 1.4

Solve the following equations.

(1) $2x+5=\sqrt{7x+16}$

Solution:

$2x+5=\sqrt{7x+16} \qquad ......(i)$

Squaring both sides, we get

$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$

$4x^2+20x+25=7x+16$

$4x^2+20x+25-7x-16=0$

$4x^2+20x-7x+25-16=0$

$4x^2+13x+9=0$

$4x^2+9x+4x+9=0$

$x(4x+9)+1(4x+9)=0$

$(x+1)(4x+9)=0$

$Either\ x+1=0\ or\ 4x+9=0$

$x=-1 \qquad 4x=-9$

$x=-\frac{9}{4}$

Check :

$Put \quad x=-1 \text{ in eq.}(i), we\ get$

$2(-1)+5=\sqrt{7(-1)+16} \qquad \Rightarrow -2+5=\sqrt{-7+16}$

$3=\sqrt{9}\Rightarrow 3=3 \ (\text{which is true})$

$Put\ x=-\frac{9}{4}\ in\ eq(i), we\ get$

$2\left(-\frac{9}{4}\right)+5=\sqrt{7\left(-\frac{9}{4}\right)+16}$

$$-\frac{9}{2}+5=\sqrt{-\frac{63}{4}+16}$$

$$\frac{1}{2}=\sqrt{\frac{1}{4}}$$

$$\frac{1}{2}=\frac{1}{2}\text{(which is true)}$$

Thus, solution set = $\left\{-1,-\frac{9}{4}\right\}$

(2) $\sqrt{x+3}=3x-1$

Solution:

$\sqrt{x+3}=3x-1 \text{ ......}(i)$

Squaring both sides, we get

$$\left(\sqrt{x+3}\right)^2=(3x-1)^2$$

$$x+3=9x^2-6x+1$$

$$9x^2-6x+1-x-3=0$$

$$9x^2-7x-2=0$$

$$9x^2-9x+2x-2=0$$

$$9x(x-1)+2(x-1)=0$$

$$(9x+2)(x-1)=0$$

Either $9x+2=0$ *or* $x-1=0$

$$9x=-2 \qquad x=1$$

$$x=-\frac{2}{9} \qquad x=1$$

Check :

Put $x=-\frac{2}{9}$ in eq.(i), *we get*

$$\sqrt{-\frac{2}{9}+3}=3\left(-\frac{2}{9}\right)-1$$

$$\sqrt{\frac{25}{9}}=-\frac{2}{3}-1$$

$$\sqrt{\frac{25}{9}} = -\frac{5}{3}$$

$$\frac{5}{3} \neq -\frac{5}{3} \text{ (which is not true)}$$

Put $x = 1$ *in* $eq(i)$, *we get*

$$\sqrt{1+3} = 3(1) - 1$$

$$\sqrt{4} = 3 - 1$$

$$2 = 2 \text{ (which is true)}$$

Thus, solution set = $\{1\}$

(3) $4x = \sqrt{13x+14} - 3$

Solution:

$$4x = \sqrt{13x+14} - 3 \qquad ......(i)$$

$$4x + 3 = \sqrt{13x+14}$$

Squaring both sides, we get

$$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$$

$$16x^2 + 24x + 9 = 13x + 14$$

$$16x^2 + 24x - 13x + 9 - 14 = 0$$

$$16x^2 + 11x - 5 = 0$$

$$16x^2 + 16x - 5x - 5 = 0$$

$$16x(x+1) - 5(x+1) = 0$$

$$(16x-5)(x+1) = 0$$

Either $16x - 5 = 0$ *or* $x + 1 = 0$

$$16x = 5 \qquad x = -1$$

$$x = \frac{5}{16} \qquad x = 1$$

Check :

Put $x = \frac{5}{16}$ in eq.(i), *we get*

$$4\left(\frac{5}{16}\right)=\sqrt{13\left(\frac{5}{16}\right)+14-3} \qquad \frac{5}{4}=\sqrt{\frac{65}{16}+14}-3$$

$$\frac{5}{4}=\sqrt{\frac{289}{16}}-3 \qquad \frac{5}{4}=\frac{17}{4}-3$$

$$\frac{5}{4}=\frac{5}{4} \quad \text{(which is true)}$$

$Put\; x=-1 \;\; in\, eq(i), we\, get$

$$4(-1)=\sqrt{13(-1)+14}-3 \qquad -4=\sqrt{-13+14}-3$$

$$-4=\sqrt{1}-3 \qquad -4=1-3$$

$$-4\neq-2 \quad \text{(which is not true)}$$

Thus, solution set = $\left\{\frac{5}{16}\right\}$

(4) $\sqrt{3x+100}-x=4$

Solution:

$$\sqrt{3x+100}-x=4 \qquad ......(i)$$

$$\sqrt{3x+100}=4+x$$

Squaring both sides, we get

$$\left(\sqrt{3x+100}\right)=^{2}(x+4)^{2}$$

$$3x+100=x^2+8x+16$$

$$x^2+8x+16-3x-100=0$$

$$x^2+5x-84=0$$

$$x^2+12x-7x-84=0$$

$$x(x+12)-7(x+12)=0$$

$$(x-7)(x+12)=0$$

$Either\;\; x-7=0 \;\; or \;\; x+12=0$

$$x=7 \qquad\qquad x=-12$$

$Check:$

$Put \quad x=7$ in eq.(i), $we\, get$

$\sqrt{13(7)+100}-7=4 \qquad \Rightarrow \sqrt{21+100}-7=4$

$\sqrt{121}-7=4 \qquad \Rightarrow 11-7=4$

$4=4$ (which is true)

$Put\ x=-12\ \ in\, eq(i), we\ get$

$\sqrt{3(-12)+100}-(-12)=4 \qquad \sqrt{-36+100}+12=4$

$\sqrt{64}=12=4 \qquad 8+12=4$

$20\neq 4$ (which is not true)

Thus, solution set = $\{7\}$

(5) $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution:

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60} \qquad ......(i)$

Squaring both sides, we get

$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$

$(x+5)+(x+21)+2\sqrt{(x+5)+(x+21)}=x+60$

$x+5+x+21+2\sqrt{x^2+26x+105}=x+60$

$2x+26+2\sqrt{x^2+26x+105}=x+60$

$2\sqrt{x^2+26x+105}=x+60-2x-26$

$2\sqrt{x^2+26x+105}=-x+34$

$2\sqrt{x^2+26x+105}=-(x-34)$

Squaring both sides, we get

$\left(2\sqrt{x^2+26x+105}\right)^2=\left[-(x-34)\right]^2$

$4\left(x^2+26x+105\right)=x^2-68x+1156$

$4x^2+104x+420=x^2-68x+1156$

$4x^2-x^2+104x+68x+420-1156=0$

$3x^2+172x-736=0$

Here a=3, b=172, c=-736

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$

$$x=\frac{-172\pm\sqrt{29584+8832}}{6}$$

$$x=\frac{-172\pm\sqrt{38416}}{6}$$

$$x=\frac{-172-196}{6} \quad or \quad x=\frac{-172+196}{6}$$

$$x=-\frac{368}{6} \qquad x=\frac{24}{6}$$

$$x=-\frac{184}{3} \qquad x=4$$

Check :

Put $x=-\frac{184}{3}$ in eq.(i), *we get*

$$\sqrt{-\frac{184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{\frac{-184}{3}+60}$$

$$\sqrt{-\frac{169}{3}}+\sqrt{-\frac{121}{3}}=\sqrt{-\frac{4}{3}} \qquad \text{(which is not true)}$$

Put $x=4$ *in eq* (i), *we get*

$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$

$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$

$$3+5=8$$

$$8=8 \qquad \text{(which is true)}$$

Thus, solution set = $\{8\}$

(6) $\sqrt{x-1}+\sqrt{x-2}+\sqrt{x+6}$

Solution:

$$\sqrt{x-1}+\sqrt{x-2}+\sqrt{x+6} \qquad ......(i)$$

Squaring both sides, we get

$$\left(\sqrt{x-1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$

$$(x+1)+(x-2)+2\sqrt{(x+1)+(x-2)}=x+6$$

$$x+1+x-2+2\sqrt{x^2-x-2}=x+6$$

$$2x-1+2\sqrt{x^2-x-2}=x+6$$

$$2\sqrt{x^2-x-2}=-x+7$$

$$2\sqrt{x^2-x-2}=-(x-7)$$

Squaring both sides, we get

$$\left(2\sqrt{x^2-x-2}\right)^2=\left[-(x-7)\right]^2$$

$$4\left(x^2-x-2\right)=x^2-14x+49$$

$$4x^2-4x-8=x^2-14x+49$$

$$4x^2-x^2-4x+14x-8-49=0$$

$$3x^2+10x-57=0$$

Here a=3, b=10, c=-57

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-10\pm\sqrt{(10)^2-4(3)(-57)}}{2(3)}$$

$$x=\frac{-10\pm\sqrt{100+684}}{6}$$

$$x=\frac{-10\pm\sqrt{784}}{6}$$

$$x=\frac{-10\pm 28}{6}$$

$$x=\frac{-10-28}{6}\quad or \qquad x=\frac{-10+28}{6}$$

$$x=\frac{-38}{6} \qquad\qquad x=\frac{18}{6}$$

$$x=-\frac{19}{3} \qquad\qquad x=3$$

Check :

Put $x=-\frac{19}{3}$ in eq.(i), *we get*

$\sqrt{-\frac{19}{3}}+1\sqrt{\frac{-19}{3}-2}=\sqrt{-\frac{19}{3}+6}$

$\sqrt{-\frac{16}{3}}+\sqrt{\frac{-25}{3}}=\sqrt{-\frac{1}{3}}$ (which is not true)

Put $x=3$ *in eq* (i), *we get*

$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$

$\sqrt{4}+\sqrt{1}=\sqrt{9}$

$2+1=3$

$3=3$ (which is true)

Thus, solution set = $\{3\}$

(7) $\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$

Solution:

$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$ ......(i)

Squaring both sides, we get

$\left(\sqrt{11-x}+\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$

$(11-x)+(6-x)+2\sqrt{(11-x)+(6-x)}=27-x$

$11-x+6-x+2\sqrt{(11-x)+(6-x)}=27-x$

$17-2x+2\sqrt{x^2-17x+66}=27-x$

$2\sqrt{x^2-17x+66}=27-x-17+2x$

$2\sqrt{x^2-17x+66}=10+x$

Squaring both sides, we get

$\left(2\sqrt{x^2-17x+66}\right)^2=(10+x)^2$

$4\left(x^2-17x+66\right)=100+20x+x^2$

$$4x^2-68x+264=x^2+20x+100$$
$$4x^2-x^2-68x+20x+264-100=0$$
$$3x^2-88x+164=0$$

Here a=3, b=-88, c=164

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$$
$$x=\frac{88\pm\sqrt{7744+1968}}{6}$$
$$x=\frac{88\pm\sqrt{5776}}{6}$$
$$x=\frac{88\pm 76}{6}$$
$$x=\frac{88-76}{6}\ or\ x=\frac{88+76}{6}$$
$$x=\frac{12}{6}\qquad x=\frac{164}{6}$$
$$x=2\qquad x=\frac{82}{3}$$

Check :

Put $x=2$ in eq.(i), *we get*

$$\sqrt{11-2}+\sqrt{6-2}=\sqrt{27-2}$$
$$\sqrt{9}+\sqrt{4}=\sqrt{25}$$
$$3+2=5$$
$$5=5\text{(which is true)}$$

Put $x=\frac{82}{3}$ *in eq* (i), *we get*

$$\sqrt{11-\frac{82}{3}}+\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$$
$$\sqrt{-\frac{49}{3}}+\sqrt{-\frac{64}{3}}$$
$$=\sqrt{-\frac{1}{3}}\text{ (which is not true)}$$

Thus, solution set = $\{2\}$

(8) $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution:

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a} \qquad ......(i)$$

Squaring both sides, we get

$$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$$

$$(4a+x)-(a-x)-2\sqrt{(4a+x)(a-x)}=a$$

$$4a+x-a+x-2\sqrt{4a^2-3ax-x^2}=a$$

$$3a+2x-2\sqrt{4a^2-3ax-x^2}=a$$

$$-2\sqrt{4a^2-3ax-x^2}=a-3a-2x$$

$$-2\sqrt{4a^2-3ax-x^2}=-2a-2x$$

$$-2\sqrt{4a^2-3ax-x^2}=-2(a+x)$$

$$\Rightarrow \sqrt{4a^2-3ax-x^2}=(a+x)$$

Squaring both sides, we get

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(a+x)^2$$

$$4a^2-3ax-x^2=(a+x)^2$$

$$4a^2-3ax-x^2=a^2+x^2+2ax$$

$$-x^2-x^2-3ax-2ax+4a^2-a^2=0$$

$$-2x^2-5ax+3a^2=0$$

$$-\left(2x^2+5ax-3a^2\right)=0$$

$$\Rightarrow 2x^2+5ax-3a^2=0$$

$$2x^2+6ax-ax-3a^2=0$$

$$2x(x+3a)-a(x+3a)=0$$

$$(2x-a)(x+3a)=0$$

$$\textit{Either } 2x-a=0 \ \textit{ or } \ x+3a=0$$

$$2x=a \qquad\qquad x=-3a$$

$$x=\frac{a}{2}$$

Thus, solution set = $\left\{-3a,\frac{a}{2}\right\}$

(9) $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution:

$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1 \quad ......(i)$

Let $x^2+x=y$

So eq.(i) becomes

$\sqrt{y+1}-\sqrt{y-1}=1$

Squaring both sides, we get

$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=1$

$(y+1)+(y-1)-2\sqrt{(y+1)(y-1)}=1$

$y+1+y-1-2\sqrt{y^2-1}=1$

$2y-2\sqrt{y^2-1}=1$

$-2\sqrt{y^2-1}=1-2y$

Squaring both sides, we get

$\left(-2\sqrt{y^2-1}\right)^2=(1-2y)^2$

$4\left(y^2-1\right)=1-4y+4y+4y^2$

$4y^2-4=1-4y+4y^2$

$4y-5=0$

$y=\frac{5}{4}$

Put $y=\frac{5}{4} \, in \, x^2+x=y, we \, get$

$4y^2-4-1+4y-4y^2=0$

$x^2+x=\frac{5}{4}$

$4x^2+4x=5$

$4x^2+4x-5=0$

Here a=4, b=4, c=-5

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm4\sqrt{6}}{8}=\frac{4\left(-1\pm\sqrt{6}\right)}{8}=\frac{-1\pm\sqrt{6}}{2}$$

(10) $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution:

$$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3 \qquad ......(i)$$

$Let\ x^2+3x=y$

So eq.(i) becomes

$$\sqrt{y+8}+\sqrt{y+2}=3$$

Squaring both sides, we get

$$\left(\sqrt{y+8}+\sqrt{y+2}\right)^2=9$$

$$(y+8)+(y+2)+2\sqrt{(y+8)(y+2)}=9$$

$$y+8+y+2+2\sqrt{y^2+10y+16}=9$$

$$2y+10+2\sqrt{y^2+10y+16}=9$$

$$2\sqrt{y^2+10y+16}=9-2y-10$$

$$2\sqrt{y^2+10y+16}=-2y-1$$

$$2\sqrt{y^2+10y+16}=-(2y+1)$$

Squaring both sides, we get

$$\left(2\sqrt{y^2+10y+16}\right)^2=\left[-(2y+1)\right]^2$$

$$4\left(y^2+10y+16\right)=4y^2+4y+1$$

$$4y^2+40y+64=4y^2+4y+1$$

$$4y^2 - 4y^2 + 40y - 4y + 64 - 1 = 0$$
$$36y = -63$$
$$y = -\frac{63}{36}$$

$Put \;\; y = -\frac{63}{36} in\, x^2 + 3x = y, we\, get$

$$x^2 + 3x = -\frac{63}{36}$$
$$\Rightarrow 36x^2 + 108x = -63$$
$$36x^2 + 108x + 63 = 0$$

Here a=36, b=108, c=63

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-108 \pm \sqrt{(108)^2 - 4(36)(63)}}{2(36)}$$
$$x = \frac{-108 \pm \sqrt{11664 + 9072}}{72}$$
$$x = \frac{-108 \pm \sqrt{2592}}{72}$$
$$x = \frac{-108 \pm 36\sqrt{2}}{72}$$
$$x = \frac{36\left(-3 \pm \sqrt{2}\right)}{72}$$
$$x = \frac{-3 \pm \sqrt{2}}{2}$$

Thus, Solution set = $\left\{\frac{-3 \pm \sqrt{2}}{2}\right\}$

(11) $\sqrt{x^2 + 3x + 9} + \sqrt{x^2 + 3x + 4} = 5$

Solution:

$$\sqrt{x^2 + 3x + 9} + \sqrt{x^2 + 3x + 4} = 5 \qquad ......(i)$$

$Let\ x^2+3x=y$

So eq.(i) becomes

$\sqrt{y+9}+\sqrt{y+1}=5$

Squaring both sides, we get

$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=25$

$(y+9)+(y+4)+2\sqrt{(y+9)(y+4)}=25$

$y+9+y+4+2\sqrt{y^2+13y+36}=25$

$2y+13+2\sqrt{y^2+13y+36}=25$

$2\sqrt{y^2+13y+36}=25-2y-13$

$2\sqrt{y^2+13y+36}=-2y+12$

$2\sqrt{y^2+13y+36}=-2(y-6)$

$\Rightarrow \sqrt{y^2+13y+36}=-(y-6)$

Squaring both sides, we get

$\left(\sqrt{y^2+13y+36}\right)^2=\left[-(y-6)\right]^2$

$y^2+13y+36=y^2-12y+36$

$y^2-y^2+13y+12y+36-36=0$

$25y=0$

$\Rightarrow y=0$

$Put\ \ y=0\ in\, x^2+3x=y, we\, get$

$x^2+3x=y$

$x^2+3x=0$

$x(x+3)=0$

$Either\, x=0\ or\ \ x+3=0$

Thus, Solution set = $\{-3,0\}$

Unit 2

Real and Complex Numbers

Exercise 2.1

Q1. Identify which of the following are rational and irrational numbers.

(1) $\sqrt{3}$ (2) $\frac{1}{3}$ (3) π (4) $\frac{15}{2}$ (5) 7.25 (6) $\sqrt{29}$

Solution:

Rational Numbers

All numbers of the form $\frac{p}{q}$ (where p, q are integers and q is not zero) are called rational numbers. The set of rational numbers is denoted by Q,

i.e., $Q' = \left\{\frac{x}{x} \neq \frac{p}{q}, p,q \in Z \wedge q \neq 0\right\}$

Irrational Numbers

The numbers which cannot be expressed as quotient of integers are called irrational numbers.

The set of irrational numbers is donated by Q'

i.e., $Q' = \left\{\frac{x}{x} \neq \frac{p}{q}, p,q \in Z \wedge q \neq 0\right\}$

For example, the numbers $\sqrt{2}, \sqrt{3}, \sqrt{5}$ are all irrational numbers. The union of the set of rational numbers and irrational numbers is known as the set of real numbers. It is denoted by R,

i.e., R = $Q \cup Q'$

here Q and Q' are both subset of R and $Q \cap Q' = \phi$

Rational Number	**2,4,5**
Irrational Numbers	**1,3,6**

Q2. Convert the following fraction into decimal fraction.

(1) $\frac{17}{25}$

Solution:

	0.68
25	170
	150
	200
	200
	0

So, $\frac{17}{25} = 0.68$ **Answer**

(2) $\frac{19}{4}$

Solution:

	4.75
4	19
	16
	30
	28
	20
	20
	0

So, $\frac{19}{4} = 4.75$ **Answer**

(3) $\frac{57}{8}$

Solution:

$$
\begin{array}{r|l}
 & \quad 7.125 \\
\hline
8 & \quad 57 \\
 & \quad 56 \\
\hline
 & \quad 10 \\
 & \quad 08 \\
\hline
 & \quad 20 \\
 & \quad 16 \\
\hline
 & \quad 40 \\
 & \quad 40 \\
\hline
 & \quad 0
\end{array}
$$

So, $\frac{57}{8} = 7.125$ **Answer**

(4) $\frac{205}{18}$

Solution:

$$
\begin{array}{r|l}
 & \quad 11.3889 \\
\hline
18 & \quad 205 \\
 & \quad 198 \\
\hline
 & \quad 70 \\
 & \quad 54 \\
\hline
 & \quad 160 \\
 & \quad 144 \\
\hline
 & \quad 160 \\
 & \quad 144 \\
\hline
 & \quad 160
\end{array}
$$

So, $\frac{205}{18} = 11.3889$ **Answer**

(5) $\frac{5}{8}$

Solution:

$$\begin{array}{r|c} & 0.625 \\ \hline 8 & 50 \\ & 48 \\ \hline & 20 \\ & 16 \\ \hline & 40 \\ & 40 \\ \hline & 0 \end{array}$$

So, $\frac{5}{8} = 0.625$ **Answer**

(6) $\frac{25}{38}$

Solution:

$$\begin{array}{r|c} & 0.65789 \\ \hline 38 & 250 \\ & 228 \\ \hline & 220 \\ & 190 \\ \hline & 300 \\ & 266 \\ \hline & 340 \\ & 304 \\ \hline & 360 \\ & 342 \\ \hline & 18 \end{array}$$

So, $\frac{25}{38} = 0.65789$

Q3. Which of the following statements are true and which are false?

(i) $\frac{2}{3}$ **is an irrational number.**

(ii) π **is an irrational number.**

(iii) $\frac{1}{9}$ **is a terminating fraction.**

(iv) $\frac{3}{4}$ **is a terminating fraction.**

(v) $\frac{4}{5}$ **is recurring fraction.**

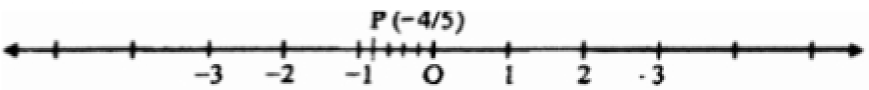
Solution:

(i) False (ii) True (iii) False (iv) True (v) False

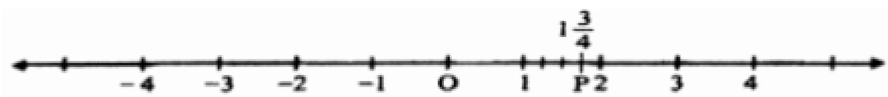
Q4. Represent the following numbers on the number line.

(1) $\frac{2}{3}$

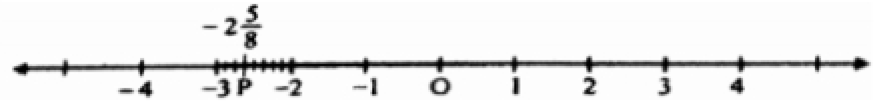
(2) $-\frac{4}{5}$



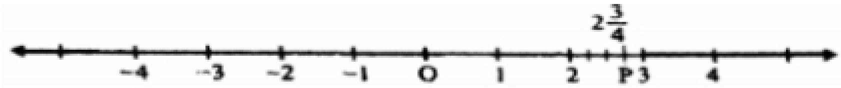
(3) $1\frac{3}{4}$



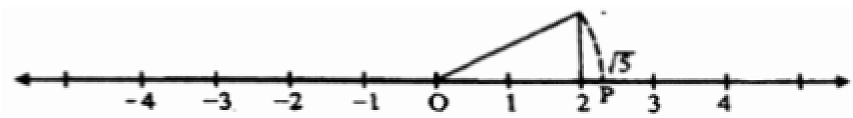
(4) $-2\frac{5}{8}$



(5) $2\frac{3}{4}$



(6) $\sqrt{5}$



Q5: Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$.

Solution:

Sum of numbers = $\frac{3}{4}$ and $\frac{5}{9} = \frac{\left(\frac{47}{36}\right)}{2} = \frac{47}{36} \times \frac{1}{2} = \frac{47}{72}$

Q6: Express the following recurring decimals as the rational number $\frac{p}{q}$ where p,q are integers and $q \neq 0$

(1) $0.\bar{5}$ **(2)** $0.\overline{13}$ **(3)** $0.\overline{67}$

Solution:

(1) $0.\bar{5}$

Let $x = 0.\bar{5}$

Or   x=0.5555....   .... (1)

Since we have only one digit I.e., 5 repeating indefinitely therefore multiplying both sides by 10

10x = 5.555....   .... (2)

Subtracting (1) from (2), we get

$10x - x = (5.555.....) - (0.555.....)$

$9x = 5.00000$

Hence $x = \frac{5}{9}$

(2) $0.\overline{13}$

Let $x = 0.\overline{13}$

Or $x = 0.131313131313....$ .... (1)

Since we have only two digits I.e., 13 repeating indefinitely therefore multiplying both side by 100

$100x = 13.1313131313....$ .... (2)

Subtracting (1) from (2), we get

$100x - x = (13.1313...) - (0.1313...)$

$99x = 13.00000$

Hence $x = \frac{13}{99}$

(3) $0.\overline{67}$

Let $x = 0.\overline{67}$

$x = 0.67676767....$ .... (1)

Since we have only two digits i.e. 67 repeating indefinitely therefore multiplying both sides by 100

$100x = 67.676767....$ .... (2)

Subtracting (1) from (2), we get

$$100x - x = (67.676767\ldots) - (0.676767\ldots)$$

$$99x = 67.00000$$

Hence $x = \frac{67}{99}$

Exercise 2.2

Q1. identify the property used in the following

i. $a+b=b+a$

ii. $a(bc)-(ab)c$

iii. $7\times1=7$

iv. $x>y, x=y, x<y$

v. $ab=ba$

vi. $a+c=b+c \Rightarrow a=b$

vii. $5+(-5)=0$

viii. $7\times\frac{1}{7}=1$

ix. $a>b \Rightarrow ac>bc \quad (c>0)$

Solution:

i. Commutative property w.r t. addition
ii. Associative property w.r.t multiplication
iii. Multiplicative identity
iv. Trichotomy property
v. Commutative property w.r.t. multiplication
vi. Cancellation property
vii. Additive inverse
viii. Multiplicative inverse
ix. Multiplicative property

Q2. Fill in the following blanks by stating the properties of real numbers used.

3x+ 3(y-x)

= 3x + 3y - 3x, ...

= 3x - 3x + 3y, ...

= 0 + 3y, ...

= 3y, ...

Solution:

3x + 3(y-x)

Step 1:

= 3x + 3y -3x Distributive property w.r.t. multiplication

Step 2:

=3x - 3x +3y, Commutative property

Step 3:

= 0+ 3y, Additive Inverse

Step 4:

= 3y, Additive identity

Q3. Give the name of property used in the following.

1. $\sqrt{24}+0=\sqrt{24}$
2. $-\frac{2}{3}\left(5+\frac{7}{2}\right)=\left(-\frac{2}{3}\right)(5)+\left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)$
3. $\pi+(-\pi)=0$
4. $\sqrt{3}.\sqrt{3}$ **is a real number**
5. $\left(-\frac{5}{8}\right)\left(-\frac{8}{5}\right)=1$

Solution:

1. Additive identity
2. Distributive property w.r.t. multiplication
3. Additive inverse
4. Closure property
5. Multiplication inverse

Exercise 2.3

Q1. Write each radical expression in exponential notation and each exponential expression in notation and each exponential expression in radical notation. Do not simplify.

Solution:

(1) $\sqrt{64} \quad = 64^{\frac{1}{2}}$

(2) $2^{\frac{3}{5}} \quad = (2^3)^{\frac{1}{5}} \quad = \sqrt[5]{2^3}$

(3) $-7^{\frac{1}{3}} \quad = \sqrt[3]{-7} \quad = -\sqrt[3]{7}$

(4) $y^{\frac{-2}{3}} \quad = (y^{-2})^{\frac{1}{3}} \quad = \sqrt[3]{y^{-2}}$

Q2. Tell whether the following statements are true or false?

(1) $5^{\frac{1}{5}} \quad = \sqrt{5}$
(2) $2^{\frac{2}{3}} \quad = \sqrt[3]{4}$
(3) $\sqrt{49} \quad = \sqrt{7}$
(4) $\sqrt[3]{x^{27}} \quad = x^3$

Solution:
(1) False
(2) True
(3) False
(4) False

Q3. Simplify the following radical expression.

(1) $\sqrt[3]{-125}$

Solution:

$= \sqrt[3]{-125}$

$= \sqrt[3]{(-5)^3}$

$= (-5)^{3\times\frac{1}{3}}$

$= -5$

(2) $\sqrt[4]{32}$

Solution:

$= \sqrt[4]{2^5}$

$= \sqrt[4]{2.2^4}$

$= \sqrt[4]{2}.\sqrt[4]{2^4}$

$= \sqrt[4]{2}.2^{4\times\frac{1}{4}}$

$= \sqrt[4]{2}.2$

$= 2\sqrt[4]{2}$

(3) $\sqrt[5]{\frac{3}{32}}$

Solution:

$= \sqrt[5]{\frac{3}{32}}$

$= \frac{\sqrt[5]{3}}{\sqrt[5]{32}}$

$= \frac{\sqrt[5]{3}}{\sqrt[5]{2^5}}$

$= \frac{\sqrt[5]{3}}{2^{5\times\frac{1}{5}}}$

$= \frac{\sqrt[5]{3}}{2}$

(4) $\sqrt[3]{-\frac{8}{27}}$

Solution:

$= \sqrt[3]{-\frac{8}{27}}$

$= \sqrt[3]{\left(-\frac{8}{27}\right)^{3}}$

$= \left(-\frac{2}{3}\right)^{3\times\frac{1}{3}}$

$= -\frac{2}{3}$

Exercise 2.4

Q1. Use laws of exponents to simplify:

(1) $\dfrac{(243)^{\frac{-2}{3}}(32)^{\frac{-1}{5}}}{\sqrt{(192)^{-1}}}$

Solution:

$$=\frac{(243)^{\frac{-2}{3}}(32)^{\frac{-1}{5}}}{(196)^{\frac{-1}{2}}}$$

$$=\left(\frac{1}{243}\right)^{\frac{2}{3}}\times\left(\frac{1}{32}\right)^{\frac{1}{5}}\times(196)^{\frac{1}{2}}$$

$$=\left(\frac{1}{3^5}\right)^{\frac{2}{3}}\times\left(\frac{1}{2^5}\right)^{\frac{1}{5}}\times(4\times49)^{\frac{1}{2}}$$

$$=\frac{1}{3^{\frac{10}{3}}}\times\frac{1}{2^{\frac{5}{5}}}\times(2^2)^{\frac{1}{2}}\times(7^2)^{\frac{1}{2}}$$

$$=\frac{1}{3^{\frac{1}{3}}\times3^{\frac{9}{3}}}\times\frac{1}{2}\times2\times7$$

$$=\frac{1}{3^3\times3^{\frac{1}{3}}}\times7$$

$$=\frac{7}{3^3\times\sqrt[3]{3}}$$

$$=\frac{7}{27(\sqrt[3]{3})}$$

(2) $(2x^5y^{-4})(-8x^{-3}y^2)$

Solution:

$$=2\times(-8)\times x^5.x^{-3}.y^{-4}.y^2$$

$$=-16x^{5-3}.y^{-4+2}$$

$$=-16x^2y^{-2}$$

$$=-\frac{16x^2}{y^2}$$

(3) $\left(\frac{x^{-2}y^{-3}z^0}{x^4y^{-3}z^0}\right)$

Solution:

$$=\left(\frac{x^4y^{-3}z^0}{x^{-2}y^{-1}z^{-4}}\right)^3$$

$$=\left(\frac{x^{4+2}.z^{0+4}}{y^{3-1}}\right)^3$$

$$=\left(\frac{x^6.z^4}{y^2}\right)$$

$$=\frac{x^{6\times3}.z^{4\times3}}{y^{2\times3}}$$

$$=\frac{x^{18}z^{12}}{y^6}$$

(4) $\frac{(81)^n.3^5-(3)^{4n-1}.(243)}{(9^{2n})(3^3)}$

Solution:

$$=\frac{(3^4)^n.3^5-(3)^{4n-1}.3^5}{(3^2)^{2n}(3^3)}$$

$$= \frac{3^{4n}.3^5 - 3^{4n+4}}{3^{4n+3}}$$

$$= 3^{4n+4-4n-3}.(2)$$

$$= (3) \times (2)$$

$$= 6$$

Q2: Show that $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

Solution:

$$= \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$$

$$= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= x^0$$

$$= 1$$

Q3: Simplify.

(1) $\dfrac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times 60^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times 4^{\frac{-1}{3}} \times 9^{\frac{1}{4}}}$

Solution:

$$= \frac{2^{\frac{1}{3}} \times (3^3)^{\frac{1}{3}} \times (3.5.2^2)^{\frac{1}{2}}}{(2.2.2.3.3.5)^{\frac{1}{2}} \times (2^2)^{\frac{-1}{3}} \times (3^2)^{\frac{1}{4}}}$$

$$= \frac{2^{\frac{1}{3}}.3^{3\times\frac{1}{3}}.3^{\frac{1}{2}}.5^{\frac{1}{2}}.2^{2\times\frac{1}{2}}}{(2^2.3^2.5)^{\frac{1}{2}}.2^{\frac{-2}{3}}.3^{\frac{2}{4}}}$$

$$= \frac{2^{\frac{1}{3}+\frac{2}{3}}.3^{\frac{1}{2}}.5^{\frac{1}{2}}.3.2}{5^{\frac{1}{2}}.3^{\frac{1}{2}}.2.3}$$

$$= 2^1$$

$$= 2$$

(2) $\sqrt{\frac{216^{\frac{2}{3}} \times 25^{\frac{1}{2}}}{(0.04)^{\frac{-1}{2}}}}$

Solution:

$$= \sqrt{\frac{(2^3.3^3).(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-1}{2}}}}$$

$$= \sqrt{2^2.3^2.5.\left(\frac{2}{10}\right)^{2\times\frac{1}{2}}}$$

$$= \sqrt{2^2.3^2.5.\left(\frac{2}{10}\right)}$$

$$= \sqrt{2^2.3^2}$$

$$= 2.3$$

$$= 6$$

(3) $5^2 \div (5^2)^3$

Solution:

$$= 5^8 \div 5^6$$

$$= 5^{8-6}$$

$$= 5^2$$

$$= 25$$

(4) $(x^3)^2 \div x^{3^2}, x \neq 0$

Solution:

$$= x^{3\times 2} \div x^{3\times 3}$$

$$= x^6 \div x^9$$

$$= x^{6-9}$$

$$= x^{-3}$$

$$= \frac{1}{x^3}$$

Exercise 2.5

Q1. Evaluate.

(1) i^7

Solution:

$= i^6 \times i$

$= \left(i^2\right)^3 \times i$

$= (-1)^3 \times i$

$= (-1) \times -i$

$= -i$

(2) i^{50}

Solution:

$= (i^2)^{25}$

$= (-1)^{25}$

$= -1$

(3) i^{12}

Solution:

$= (i^2)^{25}$

$= (-1)^{25}$

$= -1$

(4) $(-i)^8$

Solution:

$= i^8$

$= (i^2)^4$

$= (-1)^4$

$= 1$

(5) $(-i)^5$

Solution:

$= -i^4 \times i$

$= -(i^2)^2 \times i$

$= -i$

(6) i^{27}

Solution:

$= i^{26} \times i$

$= (i^2)^{23} \times i$

$= (-1)^{23} \times i$

$= (-1) \times i$

$= -i$

Q2. Write the conjugate of the following numbers.

(1) $2 + 3i$

Solution:

$z = 2 + 3i$

$\bar{z} = 2 - 3i$

(2) $3 - 5i$

Solution:

$z = 3 - 5i$

$\bar{z} = 3+5i$

(3) $-i$

Solution:

$z = -i$

$\bar{z} = i$

(4) $-3+4i$

Solution:

$z = -3+4i$

$\bar{z} = -3-4i$

(5) $-4-i$

Solution:

$z = -4-i$

$\bar{z} = -4+i$

(6) $i-3$

Solution:

$z = i-3$

$\bar{z} = -i-3$

Q3: Write the real and imaginary parts of the following number.

Solution:

(1) $1+i$ $\operatorname{Re}(z) = 1$ $\operatorname{Im}(z) = 1$

(2) $-1+2i$ $\operatorname{Re}(z) = -1$ $\operatorname{Im}(z) = 2$

(3) $-3i+2$ $\operatorname{Re}(z) = 2$ $\operatorname{Im}(z) = -3,$

(4) $-2-2i$ $\operatorname{Re}(z) = -2$ $\operatorname{Im}(z) = -2$

(5) $-3i$ $\operatorname{Re}(z) = 0$ $\operatorname{Im}(z) = -3$

(6) $2+0i \ \mathrm{Re}(z)=2 \ \mathrm{Im}(z)=0$

Q4. Find the value of x and y if $x+iy+1=4-3i$ **.**

Solution:

$$x+iy+1=4-3i$$

$$(x+1)+iy=4-3i$$

By comparing real and imaginary part we get

x+1=4 and y=-3

x=4-1 and y=-3

x=3 and y=-3

Exercise 2.6

Q1. Identify the following statements as true or false.

(1) $\sqrt{-3}\sqrt{-3}=3$

(2) $i^{73}=-i$

(3) $i^{10}=-i$

(4) Complex conjugate of $(-6i+i^2) is (-1+6i)$

(5) Difference of a complex number z=a+b*i* and its conjugate is a real number

(6) If $(a-1)-(b+3)i=5+8i$, then $a=6$ and $b=-11$

(7) Product of a complex number and its conjugate is always a non-negative real number.

Solution:

(1) False

(2) False

(3) True

(4) True

(5) False

(6) True

(7) True

Q2. Express each complex number in the standard form a+bi, where a and b are real numbers.

Solution:

(1) (2+3i) + (7-2i)

$$=(2+7)+(3-2)i$$
$$=9+i$$

(2) 2(5+4i) - 3(7+4i)

$= 10+8i-21-12i$
$= -11-4i$

(3) -(-3+5i) - (4+9i)

$= 3-5i-4-9i$
$= -1-14i$

(4) $= 2i^2 + 6i.i^2 + 3(i^2)^8 - 6i^{18}.i + 4i^{24}.i$

$= -2+6i(-1)+3(1)-6(i^2)^9 i + 4(i^2)^{22} i$

$= -2-6i(-1)+3(1)-6(i^2)^9 i + 4(i^2)^{22} i$

$= -2-6i+3+6i+4i$

$= 1+4i$

Q3. Simplify and write your answers in the form a+bi.

Solution:
(1) (-7+3i)(-3+2i)

Solution:

$= 21-14i-9i+6i^2$

$= 21-23i+6(-1)$

$= 21-23i-6$

$= 15-23i$

(2) $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

Solution:

$= (2-2i)(3-2i)$

$= 6-4i-6i+4i^2$

$= 6-10i-4$

$$=2-10i$$

(3) $\left(\sqrt{5}-3i\right)^2$

Solution:

$$=\left(\sqrt{5}\right)^2+(3i)^2-2\left(\sqrt{5}\right)(3i)$$

$$=5+9(-1)-6\sqrt{5i}$$

$$=5-9-6\sqrt{5i}$$

$$=-4-6\sqrt{5i}$$

(4) $(2-2i)(\overline{3+2i})$

Solution:

$$=(2-3i)(3+2i)$$

$$=6+4i-9i-6i^2$$

$$=6-5i-6(-1)$$

$$=6+6-5i$$

$$=12-5i$$

Q4: Simplify and write your answer in the form a+bi.

(1) $\frac{-2}{1+i}$

$$=\frac{-2}{1+i}\times\frac{1-i}{1-i}$$

$$=\frac{-2+2i}{1+1}$$

$$=-1+i$$

(2) $\frac{2+3i}{4-i}$

$$=\frac{2+3i}{4-i}\times\frac{4+i}{4+i}$$

$$=\frac{8+2i+12i+3i^2}{16+1}$$

$$=\frac{5+4i}{17}$$

(3) $\frac{9-7i}{3+i}$

$$=\frac{9+7i}{3+i}\times\frac{3-i}{3-i}$$

$$=\frac{27-30i-7}{10}$$

$$=2-3i$$

(4) $\frac{2-6i}{3+i}\times\frac{4+i}{3+i}$

$$=\frac{2-6i-4-i}{3+i}$$

$$=\frac{-2-7i}{3+i}\times\frac{3-i}{3-i}$$

$$=\frac{-6-19i-7}{10}$$

$$=\frac{-13-19i}{10}$$

(5) $\left(\frac{1+i}{1-i}\right)^2$

$$=\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^2$$

$$=\left(\frac{1+2i+i^2}{1+1}\right)^2$$

$$=\left(\frac{2i}{2}\right)^2$$

$$=i^2$$

$$=-1$$

(6) $\frac{1}{(2+3i)(1-i)}$

$$=\frac{1}{2-2i+3i-3i^2}$$

$$=\frac{1}{5+i}$$

$$=\frac{1}{5+i}\times\frac{5-i}{5-i}$$

$$=\frac{5-i}{25+1}$$

$$=\frac{5-i}{26}$$

Q5. Calculate $(a)\bar{z}$ **(b)** $z+\bar{z}$ **(c)** $z-\bar{z}$ **(d)** $z\bar{z}$ **for each of the following**

(1) $z=-i$
(2) $z=2+i$
(3) $z=\frac{1+i}{1-i}$
(4) $z=\frac{4-3i}{2+4i}$

Solution:

(1) z=-i

$z = 0 - 1i$

(a) $z + \bar{z} = -i + i = 0$

(b) $z - \bar{z} = -i - i = -2i$

(c) $z\bar{z} = (-i)i = -i^2 = -(-1) = 1$

(2) z=2+I

(a) $\bar{z} = 2 - i$

(b) $z + \bar{z} = 2 + i + 2 - i = 4$

(c) $z - \bar{z} = 2 + i - 2 + i = 2i$

(d) $z\bar{z} = (2 + i)(2 - i) = 4 - i^2 = 4 - (-1) = 4 + 1 = 5$

(3) $z = \dfrac{1+i}{1-i}$

(a) $\bar{z} = 0 - i = -i$

(b) $z + \bar{z} = i - i = 0$

(c) $z - \bar{z} = i - (-i) = i + i = 2i$

(d) $z\bar{z} = i(-i) = i^2 = -(-1) = 1$

(4) $z = \dfrac{4-3i}{2+4i}$

$$z = \frac{4-3i}{2+4i} \times \frac{2-4i}{2-4i}$$

$$= \frac{(4-3i)(2-4i)}{4-16i^2}$$

$$= \frac{8-16i-6i+12i^2}{4-16(-1)}$$

$$= \frac{8-22i-12}{20}$$

$$= \frac{-4-22i}{20}$$

$$= -\frac{1}{5} - \frac{11}{10}i$$

(a) $\bar{z} = -\frac{1}{5} + \frac{11}{10}i$

(b) $z + \bar{z} = -\frac{1}{5} - \frac{11}{10}i - \frac{1}{5} + \frac{11}{10}i = -\frac{2}{5}$

(c) $z - \bar{z} = -\frac{1}{5} - \frac{11}{10}i - \frac{1}{5} - \frac{11}{10}i = -\frac{22}{10}i$

(d) $z\bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right)\left(-\frac{1}{5} + \frac{11}{10}i\right)$

$$= \left(-\frac{1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2$$

$$= \frac{1}{25} + \frac{121}{100}$$

$$= \frac{125}{100}$$

$$= \frac{5}{4}$$

Q6. **If** $z = 2+3i$ **and** $w = 5-4i$**, show that.**

(i) $\overline{z+w} = \bar{z} + \bar{w}$

(ii) $\overline{z-w} = \bar{z} - \bar{w}$

(iii) $\overline{zw} = \bar{z}\bar{w}$

(iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$, where $w \neq 0$.

(v) $\frac{1}{2}(z+\bar{z})$ is the real part of z

(vi) $\frac{1}{2i}(z-\bar{z})$ is the imaginary part of z

Solution:

$z = 2+3i \quad \Rightarrow \bar{z} = 2-3i$

$w = 5-4i \quad \Rightarrow \bar{w} = 5+4i$

i) $\overline{z+w} = \bar{z}+\bar{w}$

$L.H.S = \overline{z+w} = 7+i$

$R.H.S = \bar{z}+\bar{w} = 2-3i+5+4i = 7+i$

Hence

$L.H.S = R.H.S$

ii) $\overline{z-w} = \bar{z}-\bar{w}$

$L.H.S = \overline{z-w} = -3-7i$

$R.H.S = \bar{z}-\bar{w} = 2-3i-5-4i = -3i$

Hence

$L.H.S = R.H.S$

iii) $\overline{zw} = \bar{z}\bar{w}$

$\overline{zw} = \bar{z}\bar{w}$

$L.H.S = \overline{zw} = 22-7i$

$R.H.S = \bar{z}\bar{w} = (2-3i)(5+4i) = 10+8i+15i-12i^2$

$= 10-7i-12(-1) = 10+12-7i = 22-7i$

Hence

$L.H.S = R.H.S$

iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}, where, w \neq 0.$

$\overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}, where, w \neq 0.$

$$\frac{z}{w} = \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i} = \frac{10+8i+15i+12i^2}{25-16(-1)}$$

$$= \frac{-2+23i}{41} = -\frac{2}{41} + \frac{23}{41}i$$

$$L.H.S = -\frac{2}{41} - \frac{23}{41}i$$

$$R.H.S = \frac{2-3i}{5+4i} \times \frac{5-4i}{5-4i} = \frac{10-8i-15i+12i^2}{25-16(-1)}$$

$$= \frac{-2-23i}{41} = -\frac{2}{41} - \frac{23}{41}i$$

Hence

$L.H.S = R.H.S$

v) $\frac{1}{2}(z+\overline{z})$ is the real part of z

$$\frac{1}{2}(z+\overline{z}) = \frac{1}{2}(2+3i+2-3i)$$

$$= \frac{1}{2}(4) = 2$$

vi) $\frac{1}{2i}(z-\overline{z})$ is the imaginary part of z

$$\frac{1}{2i}(z-\overline{z}) = \frac{1}{2i}(2+3i-2+3i)$$

$$= \frac{1}{2i}(6i) = 3$$

Q7. Solve the following equations for real x and y.

i) $(2-3i)(x+yi)=4+i$

ii) $(3-2i)(x+yi)=2(x-2yi)+2i-1$

iii) $(3+4i)^2-2(x-yi)=x+yi$

Solution:

i) $(2-3i)(x+yi)=4+i$

$2x+2yi-3xi-3yi^2=4+i$

$2x-3y(-1)-3xi+2yi=4+i$

By comparing

$2x+3y=4 \quad \text{(i)}$

$-3x+2y=1 \quad \text{(ii)}$

Now multiplying (i) & (ii)

$6x+9y=12 \quad \text{(iii)}$

$-6x+4y=2 \quad \text{(iv)}$

Adding (iii) & (iv)

$13y=14$

$y=\frac{14}{13}$

Putting (y) in (i)

$2x+3\left(\frac{14}{13}\right)=4$

$2x=4-\frac{42}{13}$

$2x=\frac{52-42}{13}$

$2x=\frac{10}{13}$

$x=\frac{5}{3}$

Hence

$x=\frac{5}{3}$ and $y=\frac{14}{13}$

ii) $(3-2i)(x+yi)=2(x-2yi)+2i-1$

$3x+3yi-2xi-2yi^2=2x-4yi+2i-1$

$3x-2y(-1)-3yi+2xi=2x-1-4yi+2i$

By comparing

$3x+2y=2x-1$

$x+2y=-1$ (i)

$3y-2x+4y=2$

$7y-2x=2$ (ii)

Multiplying (i) & (ii), we get

$11y=0 \Rightarrow y=0$

Now substitute y=0 in (i), we get

$x+2y=-1$

$x=-1$

Hence

$x=-1 \ and \ y=0$

iii) $(3+4i)^2-2(x-yi)=x+yi$

$9+16i^2+24i-2x+2yi=x+yi$

$9-16+24i-2x+2yi=x+yi$ $\quad(\because i^2=-1)$

$(-7-2x)+(24+2y)i=x+yi$

By comparing

$-7-2x=x$

$-2x-x=7 \Rightarrow -3x=7$

$\Rightarrow x=-\frac{7}{3}$

$24+2y=y$

$2y-y=-24$

$y=-24$

Hence

$x=-\frac{7}{3}$ and $y=-24$

Exercise 2.7

Solve the following simultaneous equations.

1. $x + y = 5$; $x^2 - 2y - 14 = 0$

Solution:

$x + y = 5$ ....(i)

$x^2 - 2y - 14 = 0$ ....(ii)

From eq. (i), we have

$y = 5 - x$ ....(iii)

Put value of y in eq.(ii), we get

$x^2 - 2(5 - x) - 14 = 0$

$x^2 - 10 + 2x - 14 = 0$

$x^2 + 2x - 24 = 0$

$x^2 + 6x - 4x - 24 = 0$

$x(x + 6) - 4(x + 6) = 0$

$(x - 4)(x + 6) = 0$

Either $x - 4 = 0$ *or* $x + 6 = 0$

$x = 4$ $x = -6$

Put $x = 4$ in eq.(iii), we get

$y = 5 - 4$

$= 1$

Put $x = -6$ in eq.(iii), we get

$y = 5 - (-6)$

$= 5 + 6$

$= 11$

The ordered pairs are $(4,1)$ and $(-6,11)$.

Thus, solution set $= \{(4,1),(-6,11)\}$

2. $3x - 2y = 1$; $x^2 + xy - y^2 = 1$

Solution:

$3x-2y=1 \qquad ....(i)$

$x^2+xy-y^2=1 \qquad ....(ii)$

From $eq.\ (i)$, we have

$2y=3x-1$

$y=\frac{1}{2}(3x-1)$

$y=\frac{3}{2}x-\frac{1}{2} \qquad ....(iii)$

Put value of y in eq.(ii), we get

$x^2+x\left(\frac{3}{2}x-\frac{1}{2}\right)-\left(\frac{3}{2}x-\frac{1}{2}\right)^2=1$

$x^2+\frac{3}{2}x^2-\frac{1}{2}x-\frac{9}{4}x^2+\frac{3}{2}x-\frac{1}{4}=1$

Multiplying both sides by '4' we get

$4x^2+6x^2-2x-9x^2+6x-1=4$

$4x^2+6x^2-9x^2-2x+6x-1-4=0$

$x^2+4x-5=0$

$x^2+5x-x-5=0$

$x(x+5)-1(x+5)=0$

$(x-1)(x+5)=0$

Either $x-1=0$ *or* $x+5=0$

$x=1$ $\qquad$ $x=-5$

Put $x=1$ in eq.(iii), we get

$y=\frac{3}{2}(1)-\frac{1}{2}$

$=\frac{3}{2}-\frac{1}{2}$

$=\frac{2}{2}$

$=1$

Put $x=-5$ in eq.(iii), we get

$y=\frac{3}{2}(-5)-\frac{1}{2}$

$=-\frac{15}{2}-\frac{1}{2}$

$=-\frac{16}{2}$

$=-8$